Solving tangram puzzles with simulated annealing

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Abstract: The tangram is a dissection puzzle composed of polygonal pieces which can be combined to form different patterns. Besides being a recreational puzzle, the tangram relates to a more general class of combinatorial NP-hard problems such as the bin packing problem and jigsaw puzzles. In this paper, we present preliminary results about a computational method for solving tangram puzzles based on the simulated annealing approach.

1. Introduction

The tangram is a geometric puzzle composed by seven polygonal pieces: a square, a parallelogram, and five triangles of different sizes. In Figure 1, the pieces are presented in a standard square configuration. The goal of the puzzle is to rearrange the seven pieces using rigid body transformations in order to fit them into a given pattern composed of simply-connected or multiply-connected planar regions [4]. All seven pieces must be used and they may not overlap. Pieces may contact each other in a vertex-to-vertex, vertex-to-edge, and edge-to-edge fashion.

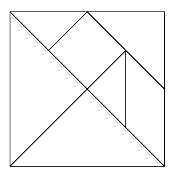


Fig. 1. Tangram pieces in the standard configuration.

Most computational puzzle solving techniques are dedicated to jigsaw [1] and edge-matching [5] puzzles. Similar to the tangram puzzle problem, jigsaw and edge-matching puzzles are known to be NP-hard problems. We have found in the literature two techniques proposed exclusively for tangram puzzles and two techniques proposed for different genres of puzzles that include extensions to tangram puzzles. After a preliminary identification of the limitations of these techniques, we have outlined a new method that aims to mitigate such current limitations. Our method is based on the simulated annealing approach [10].

2. Computational methods to solve tangram puzzles

Deutsch & Hayes [6] proposed a method for solving tangram puzzles using heuristic programming. Initially, the algorithm performs attempts and tests of partitioning a desired polygonal pattern into smaller parts called sub-puzzles. The algorithm follows the contour of the pattern and, in convex corners (from internal angles), generates extension lines that determine a possible section of the pattern. In order to separate the pattern into sub-puzzles, the method applies ten rules that consider the relation between the edges of the pattern and the extension lines with the pieces and the composites, which are regions formed by a set of pieces. The algorithm rearranges the pieces correctly for most examples presented in the original paper, but the authors present cases in which it is not possible to obtain satisfactory results. In addition, the approach is limited to patterns without holes and only allows rotations of pieces by angles at multiples of 45 degrees.

Oflazer [9] presents a computational technique to the placement of tangram pieces based on a non-restricted Boltzmann Machine. The pieces are initially laid out on a regular grid. Possible positions and rotations are represented by neural units that receive excitatory connections from input units that define the puzzle, and lateral inhibitory connections of conflicting units. Oflazer presents tests performed over ten patterns commonly used in the tangram puzzle. According to the author, the method succeeds and converges in a few hundred iterations. However, it limits the rotations of pieces by angles at multiples of 45 degrees due to the regular grid.

Bartoněk [2] presents a genetic algorithm approach for solving polygonal jigsaw puzzle. This method presents an extension for the solution of tangram puzzles in which pieces are represented by string codes. In a string code, the edges and angles are represented by integer numbers invariant to rigid body transformations. Each piece is assigned to a certain group according to the calculated similarity between the piece and the other pieces belonging to the same group based on its string codes. An evaluation function (fitness) determines how many groups the pieces will be divided into.

Kovalsky *et al.* [7] present a method for solving jigsaw puzzles in terms of algebraic concepts. The puzzle is modeled as a system of polynomial equations so that any solution of the system is a solution of the puzzle as a complete representation. The authors first propose to solve edge-matching puzzles. However, they show how to apply their approach to tangram puzzles by considering the tangram as an edge-matching puzzle in which all pieces have the same color.

Table 1 summarizes the main aspects and limitations of each approach as claimed by their authors. In our preliminary comparison, we considered four main aspects: the limitation in the rotation of pieces, the parallelogram reflection (flip) transformation, the ability of solving patterns with holes and the ability of solving patterns with multiply-connected regions.

Table 1. Preliminary comparison of computational methods to solve tangram puzzles

Authors	Rotations are limited?	Deal with parallelogram flip?	Solve patterns with holes?	Solve patterns with multiply-connected regions?
Deutsch & Hayes	✓	✓		
Oflazer	✓	✓	✓	
Bartonek	✓			
Kovalsky	✓			

According to our first analysis, the neural network method by Oflazer seems to be more flexible in the sense of solving the tangram puzzle for patterns with holes and patterns which require the parallelogram reflection (flip) transformation. However, all the aforementioned approaches show limitations with respect to piece orientation, usually limiting rotations to a discrete set of multiples of 45 degrees. In addition, none of the approaches proved to be applicable in the solution of patterns with multiply-connected regions.

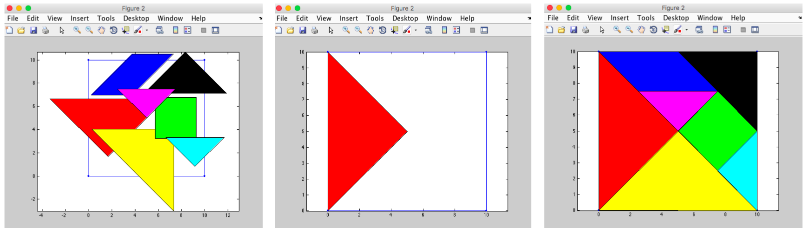
Our proposal is based on the work of Martins & Tsuzuki [8], which presents a simulated annealing approach for solving the problem of minimizing the waste of space that occurs on a rotational placement of a set of irregular bi-dimensional small items inside a bi-dimensional large object. This approach represents a feasible technique to solve simple tangram puzzles, since the authors stated that the cutting and packing problem share some similarities with the tangram patterns. Therefore, we aim to introduce some modifications to this method in order to solve patterns with the parallelogram flip, patterns with holes and patterns formed by multiply-connected regions that are more complex cases of tangram puzzles.

3. Simulated Annealing

Simulated annealing is a probabilistic global optimization technique. It consists in a hill climbing method in which better solutions are always accepted, and worse solutions may also be accepted with some probability, which allows the algorithm to jump out of a local minimum and contributes for a better exploration of the solution space [3].

In order to exemplify the application of simulated annealing to the solution of tangram puzzles, we formulated a simple task involving the pieces orientations.

Initially, all the pieces are already placed in their final position inside the pattern contour. However, each piece is randomly rotated. The simulated annealing algorithm is applied on the pieces one by one in order to find the desired orientation for fitting the piece. The simulated annealing cost function is given by the sum of pieces overlapping area, and the area of the pieces outside the contour. Therefore, the goal of the simulated annealing is to devise a set of orientations that minimize the overlap. In Figure 2, the process of obtaining the orientations is illustrated.



(a) (b) (c)

Fig. 2. Proposed method for solving tangram pieces orientations. (a) Randomly rotated tangram pieces; (b) First piece final orientation; (c) Output presenting all the final orientations.

4. Final considerations

We intend to further develop our approach for solving tangram puzzles using simulated annealing. A key reference to this purpose is the work of Martins & Tsuzuki [8], since they present similarities between tangram puzzles and the cutting and packing problem using simulated annealing. Our goal is to present modifications to the simulated annealing approach in order to handle patterns that are particular cases of tangram puzzles, such as patterns that require the parallelogram reflection (flip) transformation, patterns with holes and patterns formed by multiply-connected regions.

5. References

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