General heuristic for solving irregular cutting and packing problems through a raster representation

Fernanda Miyuki Yamada and Harlen Costa Batagelo

Center for Mathematics, Computing and Cognition
Federal University of ABC
Santo Andre, Brazil
fernanda.yamada@aluno.ufabc.edu.br
harlen.batagelo@ufabc.edu.br

Abstract: The cutting and packing problems consist of placing one set of shapes inside a set of containers in order to optimize an objective. In the industrial environment, the cutting and packing problems are recurrent and, in order to find a feasible solution, experienced workers attempt to build layouts with computer-aided design systems. In recent years, the implementations of algorithms for bin packing and knapsack problems were the most needed among the NP-hard problems, surpassing important computational problems, such as set-cover, traveling salesman and graph-coloring. In this work, we propose a general heuristic to solve placement and knapsack problems using a raster representation of shapes and container.

1. Introduction

The cutting and packing problems consist of placing one set of small objects inside a set of containers in order to optimize an objective function [12]. In the industrial environment, these problems are recurrent [8]. Examples include the metal, glass, and wood industries, where a set of items required by customer orders must be cut from larger sheets or boards [4]. Two-dimensional version of the cutting and packing problems, with irregular shapes and limited-size containers, can be classified into maximization problems and minimization problems [12].

The maximization problems aim at maximizing the use of a single container, and include the placement problems and the knapsack problems [12]. The difference between the placement problems and the knapsack problems lies in the diversity of the types of shapes: in placement problems the set of shapes is weakly heterogeneous, whereas in knapsack problems the set of shapes is strongly heterogeneous [18].

On the other hand, minimization problems consist of assigning a finite set of shapes to the least possible number of containers [12]. The minimization problems include the cutting-stock problems, and the bin packing problems [18]. Similarly to maximization problems, the difference between the cutting-stock problems and the bin packing problems lies in the diversity of the respective shape sets: in cutting-stock problems the set of shapes is weakly heterogeneous, whereas in bin packing problems the set of shapes is strongly heterogeneous [12].

In this paper, we propose a computational method to solve maximization problems with different constraints using a general heuristic in the discrete domain through a raster representation of the shapes, fit area, and cost. In the proposed method, the container and shapes are converted to binary masks, and mathematical morphology operators such as dilation, distance transform, and morphological skeleton are applied to the binary masks in order to determine overlaps and distance of shapes. Also, the values of the distance transform are combined with the values of the binary masks to estimate the cost function to be minimized. The main contribution of this paper is the combination of basic geometrical techniques applied in cutting and packing problems with the raster method to turn the problems solvable and computationally more efficient.

2. Theoretical Foundation

Toledo et al. [16] propose the dotted-board model, in which the container is represented as a grid. In the dotted-board model, the no-fit polygon is transformed into a no-fit raster, that is represented by a binary matrix. Binary variables are associated with a shape type and a grid dot, where the shapes can be positioned inside the container [13].

Mundim et al. [11] present the no-fit raster [2] and the inner-fit raster [14, 2] for free form shapes. In this work, the no-fit polygon and the inner fit polygon are converted to binary masks, and the quality of the solutions obtained for irregular cutting & packing problems using the rasters depend on the discretization resolutions used to generate the raster representations of the polygons.

MirHassani & Jalaeian Bashirzadeh [10] present a greedy randomized adaptive search procedure metaheuristic based on the no-fit raster and use the constraints of the dotted-board model defined by Toledo et al. cite[16] to guarantee that the shapes do not overlap. This method takes up to 300 seconds to solve simple problems, and Mundim et al. [12] point out that the resolution time presented by the method could be considerably reduced by using directly the information provided by the no-fit raster.

Rodrigues & Toledo [13] present a clique covering mixed-integer programming model for the irregular strip packing problem based on the dotted-board model proposed by Toledo et al. [16], in which the board is represented by a grid. The authors state that the proposed model obtained better performance than the dotted-board model for most instances and solved larger instances to optimality in comparison to other works of the literature.

Mundim et al. [12] propose a general heuristic to solve the two-dimensional version of the cutting and packing problems, with irregular shapes and limited-size containers. Also, the authors used no-fit raster and inner-fit raster concepts to prevent overlaps between shapes. The computational experiments show that the proposed heuristic improved on the best solutions available in the literature for three problems: the placement, the knapsack and the cutting-stock problems. The authors compare the proposed method concerning the automatic solution of placement and maximization problems with the techniques proposed by Valle et al. [6], Dalalah et al. [5], Fischetti & Luzzi [7], Alvarez-Valdes et al. [1], Gomes & Oliveira [8].

3. Materials and Methods

The shapes information are stored in a global data structure, whose variables concern the translation, the angle of rotation, the flip, and the order of the shapes. Those values are changed along the shapes placement process in order to properly accommodate each shape inside the container area. In addition, the container is represented as a binary mask. At the beginning of our method, we consider that the container area is represented as black pixels, and the area out of the container is represented as white pixels. During the shapes placement process, if a shape is placed on a black area, all pixels covered by that shape turn white. Therefore, after applying our method on a placement or a knapsack problem, it is expected that all of the pixels of the binary mask (or at least most of them) will be assigned as white.

In the proposed method, the shapes placement is executed by a recursive method, which receives an integer parameter representing the shape identification number, taking into consideration the order established in the global data structure that stores the shapes information. This method permits the algorithm to execute a backtracking, and look for different placements as soon as it determines that a shape placement makes the arrangement of the subsequent shapes in the container impossible. Also, since each cutting and packing problem has its own requirements concerning the angles of rotation for the shapes, in the beginning of the method, we set an array containing the possible orientations for each shape.

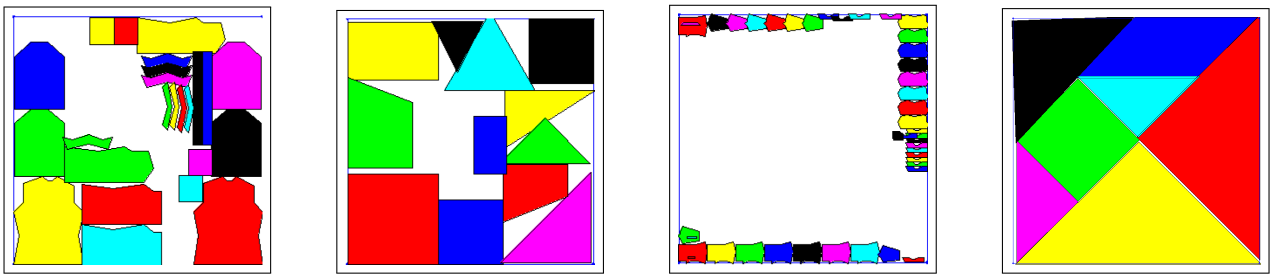
During the placement procedure, the possible angles are considered one after another until finding a feasible placement for the current shape. Thus, for each orientation, the collision-free raster areas are calculated to determine the feasible placement regions for the current shape in the current orientation [15]. Since the container mask contains the union of all the placed shapes, in order to obtain the collision-free areas, we simply dilate the container mask using the current shape in the current orientation as the structuring element of the dilation. At that point, if a collision-free area is not found, the algorithm proceeds to test the next possible angle of rotation.

If a collision-free area was found, the skeleton of each empty region of the container mask is calculated. The skeleton of a binary image is composed by a set of points whose distance from the nearest boundary of the shape is locally maximum [17]. Then, the skeleton endpoints are calculated in order to determine in which points the center of the current shape can be positioned. The placement of the shapes are executed in the endpoints because those points are more likely to accommodate the center of the shape used as the structuring element in the generation of the collision-free raster. After that, the cost of each endpoint is calculated in order to determine which of them represent the placement that has the lowest cost. The cost function takes int consideration the positioning of each pixel of the container mask, and it is obtained by applying the distance transform on the empty areas of the container mask [3]. Finally, the endpoint with the lowest cost is considered as the placement for the current shape in the current angle of rotation.

4. Results

In this section, we present some results obtained over the application of the proposed technique in instances from the ESICUP website (http://www.fe.up.pt/esicup/).

Figure 1 presents some results over the application of the proposed technique in the instances: Marques [6], Fu [6], Shirts [6], and Tangram [9].



(a) (b) (c) (d)

Fig. 1: Results of the application of the proposed techniques in instances of ESICUP. (a) Solution obtained for the Marques instance. (b) Solution obtained for the Fu instance. (c) Solution obtained for the Shirts instance. (d) Solution obtained for the Tangram instance.

5. Discussion

Considering the results obtained by applying the proposed technique on different cutting & packing problems, it is possible to observe that the technique could find a feasible solution for most of the instances. However, for the Shirts instance (Figure 1c), we could identify some overlaps between shapes inside the container. The overlaps between shapes occur due to the loss of precision that accompanies the vector-to-raster conversion. Also, the quality of the solutions obtained for irregular cutting & packing problems in the discrete domain depend on the discretization resolutions used to generate the raster representation of container and shapes.

6. Conclusion

For future works, we intend to further improve the proposed method in order to obtain a discretization resolution inversely proportional to shapes sizes aiming to assign higher resolution in the discretization of cutting & packing problems that contain smaller shapes.

In addition, we propose to implement different placement rules in order to execute tests concerning the different solutions and execution time obtained by applying each placement rule. Figure 2 present examples of paths generated by the placement rules presented in the work of Mundim et al. [12].

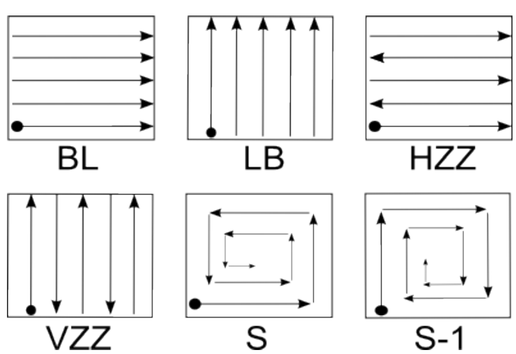


Fig. 2: Example of paths generated by different placement rules.

Finally, we intend to compare the results obtained in the tests of the proposed methods with other techniques presented in the literature for maximization problems, such as the works of Valle et al. [6], Dalalah et al. [5], Fischetti & Luzzi [7], Alvarez-Valdes et al. [1], and Gomes & Oliveira [8].

References

[1] R. Alvarez-Valdes, A. Martinez, and J. Tamarit. "A branch & bound algorithm for cutting and packing irregularly shaped pieces". In: *International Journal of Production Economics* 145.2 (2013), pp. 463–477.

[2] J. A. Bennell and X. Song. "A comprehensive and robust procedure for obtaining the nofit polygon using Minkowski sums". In: *Computers & Operations Research* 35.1 (2008), pp. 267–281.

[3] D. Brunet and D. Sills. "A Generalized Distance Transform: Theory and Applications to Weather Analysis and Forecasting". In: *IEEE Transactions on Geoscience and Remote Sensing* 55.3 (2017), pp. 1752–1764.

[4] D. J. Chalmers and B. Rabern. "Two-dimensional semantics and the nesting problem". In: *Analysis* 74.2 (2014), pp. 210–224.

[5] D. Dalalah, S. Khrais, and K. Bataineh. "Waste minimization in irregular stock cutting". In: *Journal of Manufacturing Systems* 33.1 (2014), pp. 27–40.

[6] A. M. Del Valle et al. "Heuristics for two-dimensional knapsack and cutting stock problems with items of irregular shape". In: *Expert Systems with Applications* 39.16 (2012), pp. 12589–12598.

[7] M. Fischetti and I. Luzzi. "Mixed-integer programming models for nesting problems". In: *Journal of Heuristics* 15.3 (2009), pp. 201–226.

[8] A. M. Gomes and J. F. Oliveira. "Solving irregular strip packing problems by hybridising simulated annealing and linear programming". In: *European Journal of Operational Research* 171.3 (2006), pp. 811–829.

[9] T. Martins and M. S. G. Tsuzuki. "Simulated annealing applied to the irregular rotational placement of shapes over containers with fixed dimensions". In: *Expert Systems with Applications* 37.3 (2010), pp. 1955–1972.

[10] S. MirHassani and A. J. Bashirzadeh. "A GRASP meta-heuristic for two-dimensional irregular cutting stock problem". In: *The International Journal of Advanced Manufacturing Technology* 81.1-4 (2015), pp. 455–464.

[11] L. R. Mundim, M. Andretta, and T. A. de Queiroz. "A biased random key genetic algorithm for open dimension nesting problems using no-fit raster". In: *Expert Systems with Applications* 81 (2017), pp. 358–371.

[12] L. R. Mundim et al. "A general heuristic for two-dimensional nesting problems with limited-size containers". In: *International Journal of Production Research* (2017), pp. 1–24.

[13] M. O. Rodrigues and F. M. Toledo. "A clique covering MIP model for the irregular strip packing problem". In: *Computers & Operations Research* 87 (2017), pp. 221–234.

[14] A. K. Sato, T. de Castro Martins, and M. d. S. G. Tsuzuki. "Rotational placement using simulated annealing and collision free region". In: *IFAC Proceedings Volumes* 43.4 (2010), pp. 234–239.

[15] A. K. Sato, T. C. Martins, and M. S. G. Tsuzuki. "Collision free region determination by modified polygonal Boolean operations". In: *Computer-Aided Design* 45.7 (2013), pp. 1029–1041.

[16] F. M. Toledo et al. "The dotted-board model: a new MIP model for nesting irregular shapes". In: *International Journal of Production Economics* 145.2 (2013), pp. 478–487.

[17] P. E. Trahanias. "Binary shape recognition using the morphological skeleton transform". In: *Pattern recognition* 25.11 (1992), pp. 1277–1288.

[18] G. Wäscher, H. Haußner, and H. Schumann. "An improved typology of cutting and packing problems". In: *European journal of operational research* 183.3 (2007), pp. 1109–1130.